RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4734:** Probability and Statistics 3

Mark Scheme for January 2011

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1(i)	Estμ = sample mean = 5.25	B1 **1**	
(ii)	Use (i) ± zSD SD = 0.19/√5 z = 1.96 5.083 < μ < 5.417	M1 B1 B1 A1 **4** **[5]**	 With √5 seen Rounding to 5.08, 5.42
2	Use $G – M$ ~ N(-6.23, σ^2) $\sigma^2 = 6.87^2 + 10.25^2$ z =(16.23)/ σ = 1.315 Probability = 0.0942 or 0.0943	M1 A1 M1 A1 A1 **[5]**	**Or G-M-10~ N(-16.23, σ^2)** Accept 0.094
3(i)	$\int_0^2 a\mathrm{e}^{-1}\mathrm{d}t + \int_2^\infty a\mathrm{e}^{-\frac{1}{2}t}\mathrm{d}t = 1$ $[a\mathrm{e}^{-1}t] + [-2a\mathrm{e}^{-½t}]$ => a = ¼e AG	M1 A1 A1 **3**	 Properly obtained
(ii)	$\int_{q_3}^\infty \frac{1}{4}e^{1-\frac{1}{2}t}\mathrm{d}t = \frac{1}{4}$ $[-\frac{1}{2}\mathrm{e}^{1-½t}]$ $-½q_3 + 1 = -\ln 2$ => q_3 = 2(ln2 + 1) or 3.39	M1 B1 M1 A1 **4** **[7]**	OR $\int_0^2 \frac{1}{4}\mathrm{d}t + \int_2^q \frac{1}{4}\mathrm{e}^{1-t/2}\mathrm{d}t = \frac{3}{4}$ AEF For taking logs (not ln(-)) AEF
4	$\hat{p}_2 = 106/143$, $\hat{p}_1$ = 61/107 = 0.7413 = 0.5701 Pooled est p = 167/250 Variance est = $(^{167}/_{250})(^{83}/_{250})(143^{-1}+107^{-1})$ Test statistic z = (0.7413 – 0.5701)/ SD = 2.84(35) Smallest significance level = 0.23% SR: No pe, B1B0B0M1A1(2.84)M1A1 Max 5/7	B1 B1 B1 M1 A1 M1 A1√ **[7]**	For both Only if used ART 0.22 or 0.23 Accept 0.0023 √**z** M1A0 if 0.25%
5(i)	s^2 = 0.2 × 0.8/90 $p_s \pm zs$ z = 1.645 0.1306 < p_y < 0.2693	B1 M1 B1 A1 **4**	OR /89 Art (0.131, 0.269)
(ii)	0.7306 < p_p < 0.8694	B1ft **1**	ft (i) Art (0.731, 0**.869)**
(iii)	If a large number of such intervals were calculated from independent samples, approximately 90% of all such intervals would contain p	B2 **2**	Or: Probability that such an interval contains p is 0.9 B1 for right idea
(iv)	(0.131, 0.269) encloses 0.25 so Mendel's theory is supported	M1 A1 √ **2** **[9]**	 Or equivalent Ft CI(i)

6(i)	$G(y) = P(Y \le y)$ $= P(X \ge 1/y)$ $= 1 - F(1/y)$ $= (2y - 1)/(y+1)$ For $½ \le 1/y \le 2 \Rightarrow ½ \le y \le 2$ X and Y have identical distributions SR: CDF not used. y decreases with x Use $g(y) = f(x(y)\|dx/dy)\|$ $f(x)= 3/(x+1)^2$ $\|dx/dy\|=1/y^2$ $g(y) = [3/(y^{-1}+1)^2][1/y^2] = 3/(y+1)^2$; for $½\le y\le 2$ So X and Y have identical distributions	M1 A1 M1 A1 B1 B1 **6** M1 M1A1 B1 M1A1B1 B1 **8**	 Seen
(ii)	$f(x) = F'(x) = 3/(x+1)^2$, $½\le x\le 2$ $E(X+1) = \int_{\frac{1}{2}}^{2} \frac{3}{x+1}\,dx$ $=3\ln 2$ (2.08) $E(1/X) = E(X)$ $= 3\ln 2 - 1$ (1.08)	M1A1 M1 A1 M1 A1 **6** **[12]**	Must have range of x AEF Not if awarded in (i)
7(i)	In a 2×2 contingency table	B1 **1**	Or equivalent Accept df=1
(ii)	H_0: Vaccine type and outcome are independent H_1: They are not independent E-values: 10.81 12.19 318.19 358.81 $\chi^2 = 7.69^2(10.81^{-1}+12.19^{-1}+318.19^{-1}+358.81^{-1})$ $=10.67$ CV = 6.635 **10.67** > CV Reject H_0, there is sufficient evidence at the 1% significance level that the outcome of the test depends on the vaccine used The results is significant at a level less than ½ %, so the evidence is very strong	B1M*dep M1 A1 M1 M1 A1 B1 M1 A1√ dep*M A1 √ **10** **[11]**	Accept omission of H_1 1 correct E value Accept 1 dp 1 correct χ^2 value ft E values Using Yates' correctly Accept 10.7 √ 10.67 Sensible comment.√ 10.67

8(i)	When independent samples are drawn from populations having a common variance	B1 **1**	For common variance
(ii)	**(a)** Lung capacities should have normal distributions with a common variance	B1 **1**	Normal distributions required In context here
	(b) $H_0:\mu_1=\mu_2$, $H_1:\mu_1 > \mu_2$	B1	Or equivalent
	$s_1^2 = \frac{1}{19}(90.43 - 42.4^2/20)$	M1	For 1 correct s^2
	$s_2^2 = \frac{1}{21}(82.93 - 42.5^2/22)$		
	$\bar{x}_1 = 2.12$ $\bar{x}_2 = 1.93(2)$	B1	For both
	PEV, $s^2 = (19s_1^2 + 21s_2^2)/(20+22-2)$	M1	
	$= 0.03424(3)$	A1	
	Test statistic = $\frac{2.12-1.932}{\sqrt{s^2(20^{-1}+22^{-1})}}$	M1A1	ft s^2
	= 3.29**(**15)	A1√	Accept answer rounding to 3.3
	CV = 2.423	B1	If z used the B0M0A0
	TS > CV	M1	Compare with CV
	There is sufficient evidence at the 1% SL that the mean lung capacity is greater for children whose parents do not smoke than for children whose parents do smoke	A1 **11**	Or equivalent, in context
	SR1: For 2-tail test Lose 1st B1 and last 3. Max 8/11		
	SR2: If $s^2 = s_1^2/20 + s_2^2/22$, B1M1A0A0M1A0A1(3.32) B1M1A1 Max 8/11		
	(c) $t = 2.704$	B1	
	0.1882… $\pm ts(20^{-1}+22^{-1})^{1/2}$	M1	Accept 0.19
	(0.0336, 0.3423)	A1 **3** **[16]**	(0.033-0.036,0.342-0.346)

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
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